Trigonometric Identities - Math 142

(*You can use this handout on tests and quizzes.*)

Pythagorean identities:

$$\sin^2\alpha + \cos^2\alpha = 1, \quad 1 + \cot^2\alpha = \csc^2\alpha, \quad \tan^2\alpha + 1 = \sec^2\alpha$$

Sum and Difference Identities:

$$\begin{array}{cc} \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta, & \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta, & \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \\ \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}, & \tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \end{array}$$

Product-to-sum Identities:

$$\begin{array}{cc} \sin A\cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B)), & \sin A\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B)) \\ \cos A\sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B)), & \cos A\cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B)) \\ \sin^2 A = \frac{1-\cos(2A)}{2}, & \cos^2 A = \frac{1+\cos(2A)}{2} \end{array}$$

Sum-to-product Identities:

$$\begin{array}{cc} \sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right), & \cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \\ \sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right), & \cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \end{array}$$

Law of Sines and Cosines: Let a, b and c be the sides of a triangle ABC (not necessarily a right triangle). Let θ be the angle opposite c. Then

$$c^2 = a^2 + b^2 - 2ab\cos\theta, \quad \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Two Special Identities:

$$-|\theta| \le \sin\theta \le |\theta|, \quad -|\theta| \le 1 - \cos\theta \le |\theta|$$